Cloth Simulation

COMP 768 Presentation
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at CHAPEL HILL

Outline

- Motivation and Application
- Cloth Simulation Methods
  - Physically-based Cloth Simulation
    - Overview
    - Development
- References

Motivation

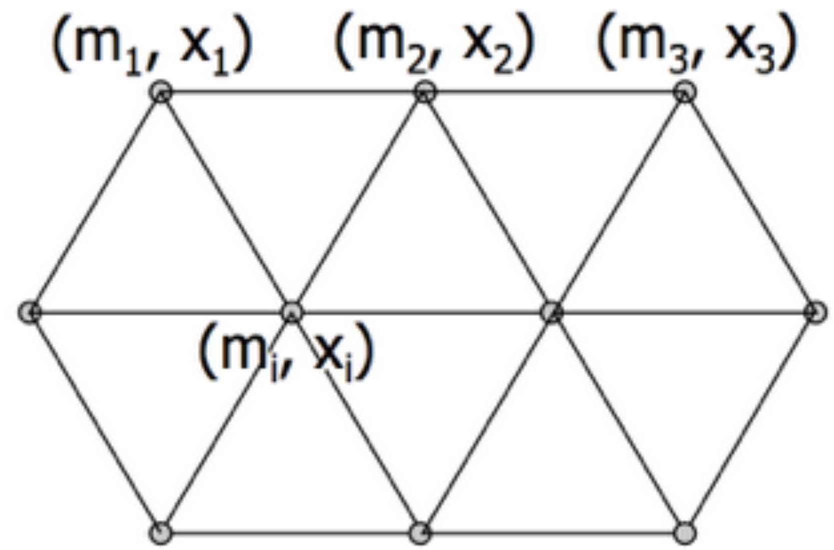
- Movies
- Games
- VR scene
- Virtual Try-on
- Fashion Design

Cloth Simulation Methods

- Geometric Method
  - Represent folds and creases by geometrical equations.
  - Aim at modeling the appearance of the cloth
  - Not focus on the physical aspects of cloth
- Physically-based Method

Physically-based Cloth Simulation

General Ideas



- Represent cloth as grids
- Vertices are points with finite mass
- Forces and energies of points are calculated from the relations with the other points

Physically-based Cloth Simulation

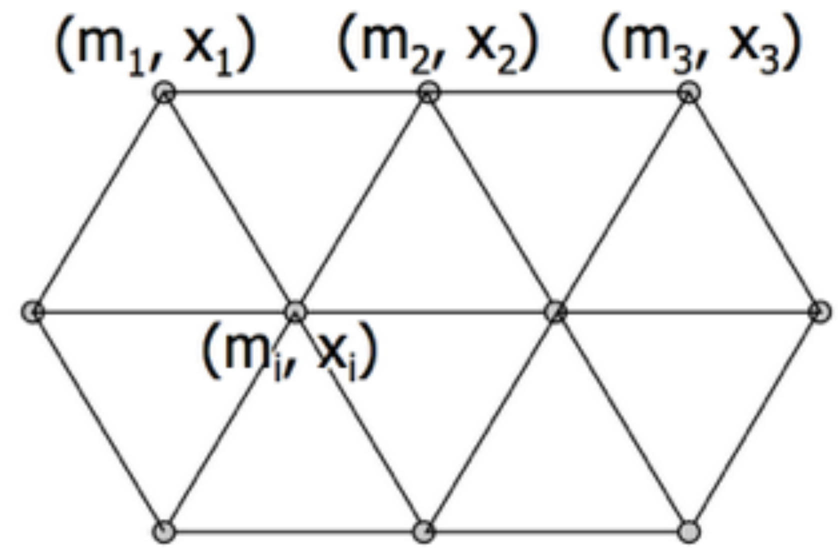
How does Cloth Simulation work

Differences among the methods

- Finding Governing Equation
- Solving the Equations
- Collision Detection / Handling

Governing Equation

$$\ddot{x} = M^{-1}(-\frac{\partial E}{\partial x} + F)$$



x : vector, the geometric state

M : diagonal matrix, mass distribution of the cloth

E : a scalar function of x, cloth's internal energy

F : a function of x and x', other forces acting on cloth

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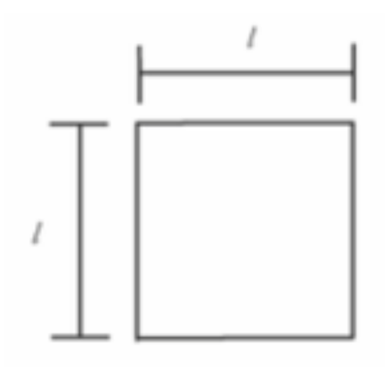
Governing Equation

- Potential energy E is related to deformations
  - Stretch
  - Shear
  - Bending
- Other Forces

$$\ddot{x} = M^{-1}\left(-\frac{\partial E}{\partial x} + F\right)$$

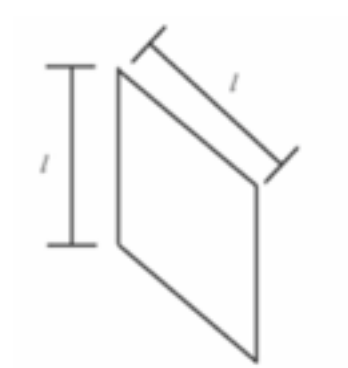


(a) Original quadrilateral of mesh

Original dihedral angle

Bending along the edge that changes dihedral angle

Figure 6.48 Control of bending by dihedral angle



b) Skew of original quadrilateral without changing the length of edges

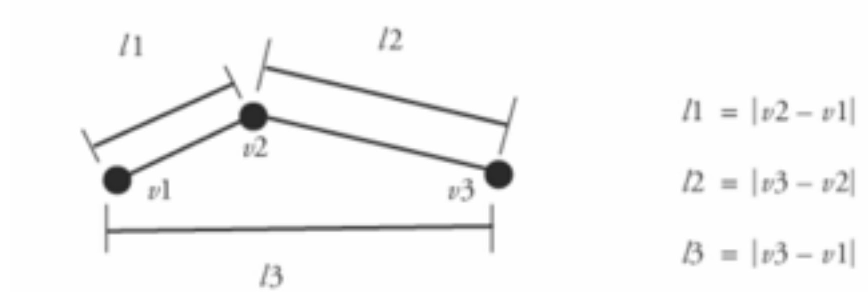


Figure 6.49 Control of bending by separation of adjacent vertices

Solving the Equations

- Explicit Euler
- Implicit Euler
- Midpoint (Leapfrog)
- Runge-Kutta
- Crank-Nicolson
- Adams-Bashforth, Adams-Moulton
- Backward Differentiation Formula (BDF)
- …

$$\ddot{x} = M^{-1}\left(-\frac{\partial E}{\partial x} + F\right)$$

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Solving the Equations

- Crank-Nicolson
  - If the partial differential equation is

$$\frac{\partial u}{\partial t} = F\left(u,\, x,\, t,\, \frac{\partial u}{\partial x},\, \frac{\partial^2 u}{\partial x^2}\right)$$

  - Solution:

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = F_i^n\left(u,\, x,\, t,\, \frac{\partial u}{\partial x},\, \frac{\partial^2 u}{\partial x^2}\right) \qquad \text{(forward Euler)}$$

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = F_i^{n+1}\left(u,\, x,\, t,\, \frac{\partial u}{\partial x},\, \frac{\partial^2 u}{\partial x^2}\right) \qquad \text{(backward Euler)}$$

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = \frac{1}{2}\left[F_i^{n+1}\left(u,\, x,\, t,\, \frac{\partial u}{\partial x},\, \frac{\partial^2 u}{\partial x^2}\right) + F_i^n\left(u,\, x,\, t,\, \frac{\partial u}{\partial x},\, \frac{\partial^2 u}{\partial x^2}\right)\right] \qquad \text{(Crank--Nicolson).}$$

$$u(i\Delta x,\, n\Delta t) = u_i^n$$

Solving the Equations

- Linear multistep method
  - Single-step methods (such as Euler's method) refer to only one previous point and its derivative to determine current value. Methods such as Runge–Kutta take some intermediate steps (for example, a half-step) to obtain a higher order method
    - Discard all previous information before taking a second step
  - Multistep methods：
    - Use the information from previous steps: refer to several previous points and derivative values to get current value.
    - Linear multistep methods：
      linear combination of the previous points and derivative values

Solving the Equations

- Adams' Method (Linear multistep method)
  - One-step Euler

$$y_{n+1} = y_n + hf(t_n, y_n).$$

$$\begin{aligned}
y_1 &= y_0 + hf(t_0, y_0) = 1 + \tfrac{1}{2} \cdot 1 = 1.5, \\
y_2 &= y_1 + hf(t_1, y_1) = 1.5 + \tfrac{1}{2} \cdot 1.5 = 2.25, \\
y_3 &= y_2 + hf(t_2, y_2) = 2.25 + \tfrac{1}{2} \cdot 2.25 = 3.375, \\
y_4 &= y_3 + hf(t_3, y_3) = 3.375 + \tfrac{1}{2} \cdot 3.375 = 5.0625.
\end{aligned}$$

  - Two-step Adams–Bashforth

$$y_{n+2} = y_{n+1} + \tfrac{3}{2} hf(t_{n+1}, y_{n+1}) - \tfrac{1}{2} hf(t_n, y_n).$$

$$\begin{aligned}
y_2 &= y_1 + \tfrac{3}{2} hf(t_1, y_1) - \tfrac{1}{2} hf(t_0, y_0) = 1.5 + \tfrac{3}{2} \cdot \tfrac{1}{2} \cdot 1.5 - \tfrac{1}{2} \cdot \tfrac{1}{2} \cdot 1 = 2.375, \\
y_3 &= y_2 + \tfrac{3}{2} hf(t_2, y_2) - \tfrac{1}{2} hf(t_1, y_1) = 2.375 + \tfrac{3}{2} \cdot \tfrac{1}{2} \cdot 2.375 - \tfrac{1}{2} \cdot \tfrac{1}{2} \cdot 1.5 = 3.7812, \\
y_4 &= y_3 + \tfrac{3}{2} hf(t_3, y_3) - \tfrac{1}{2} hf(t_2, y_2) = 3.7812 + \tfrac{3}{2} \cdot \tfrac{1}{2} \cdot 3.7812 - \tfrac{1}{2} \cdot \tfrac{1}{2} \cdot 2.375 = 6.0234.
\end{aligned}$$

Solving the Equations

- Adams' Method (Linear multistep method)
  - Adams–Bashforth methods

$$y_{n+1} = y_n + hf(t_n, y_n), \qquad \text{(This is the Euler method)}$$

$$y_{n+2} = y_{n+1} + h\left(\frac{3}{2}f(t_{n+1}, y_{n+1}) - \frac{1}{2}f(t_n, y_n)\right),$$

$$y_{n+3} = y_{n+2} + h\left(\frac{23}{12}f(t_{n+2}, y_{n+2}) - \frac{4}{3}f(t_{n+1}, y_{n+1}) + \frac{5}{12}f(t_n, y_n)\right),$$

$$y_{n+4} = y_{n+3} + h\left(\frac{55}{24}f(t_{n+3}, y_{n+3}) - \frac{59}{24}f(t_{n+2}, y_{n+2}) + \frac{37}{24}f(t_{n+1}, y_{n+1}) - \frac{3}{8}f(t_n, y_n)\right),$$

  - Adams–Moulton methods

$$y_n = y_{n-1} + hf(t_n, y_n), \qquad \text{(This is the backward Euler method)}$$

$$y_{n+1} = y_n + \frac{1}{2}h\left(f(t_{n+1}, y_{n+1}) + f(t_n, y_n)\right), \qquad \text{(This is the trapezoidal rule)}$$

$$y_{n+2} = y_{n+1} + h\left(\frac{5}{12}f(t_{n+2}, y_{n+2}) + \frac{2}{3}f(t_{n+1}, y_{n+1}) - \frac{1}{12}f(t_n, y_n)\right),$$

$$y_{n+3} = y_{n+2} + h\left(\frac{3}{8}f(t_{n+3}, y_{n+3}) + \frac{19}{24}f(t_{n+2}, y_{n+2}) - \frac{5}{24}f(t_{n+1}, y_{n+1}) + \frac{1}{24}f(t_n, y_n)\right),$$

Solving the Equations

- Backward Differentiation Formula (BDF)
  - BDF1: $y_{n+1} - y_n = hf(t_{n+1}, y_{n+1})$; (this is the backward Euler method)
  - BDF2: $y_{n+2} - \frac{4}{3}y_{n+1} + \frac{1}{3}y_n = \frac{2}{3}hf(t_{n+2}, y_{n+2})$;
  - BDF3: $y_{n+3} - \frac{18}{11}y_{n+2} + \frac{9}{11}y_{n+1} - \frac{2}{11}y_n = \frac{6}{11}hf(t_{n+3}, y_{n+3})$
  - BDF4: $y_{n+4} - \frac{48}{25}y_{n+3} + \frac{36}{25}y_{n+2} - \frac{16}{25}y_{n+1} + \frac{3}{25}y_n = \frac{12}{25}hf(t_{n+4}, y_{n+4})$
  - BDF5: $y_{n+5} - \frac{300}{137}y_{n+4} + \frac{300}{137}y_{n+3} - \frac{200}{137}y_{n+2} + \frac{75}{137}y_{n+1} - \frac{12}{137}y_n = \frac{60}{137}hf(t_{n+5}, y_{n+5})$
  - BDF6: $y_{n+6} - \frac{360}{147}y_{n+5} + \frac{450}{147}y_{n+4} - \frac{400}{147}y_{n+3} + \frac{225}{147}y_{n+2} - \frac{72}{147}y_{n+1} + \frac{10}{147}y_n = \frac{60}{147}hf(t_{n+6}, y_{n+6})$.

Methods with s > 6 are not zero-stable so they cannot be used

- vs. Adams–Moulton methods

$$y_n = y_{n-1} + hf(t_n, y_n), \qquad \text{(This is the backward Euler method)}$$

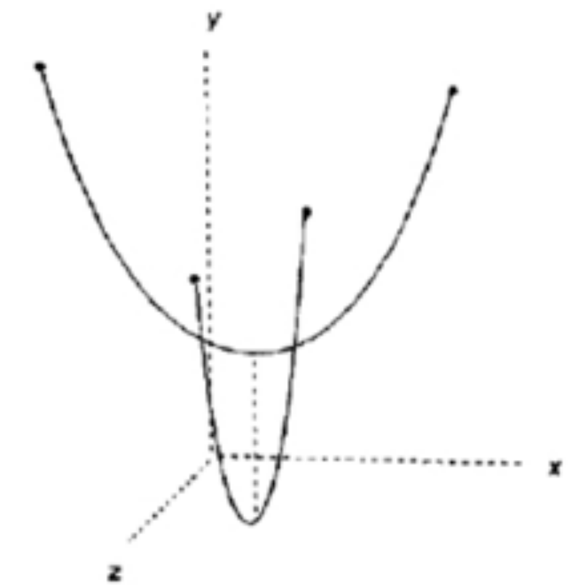
$$y_{n+1} = y_n + \frac{1}{2}h\left(f(t_{n+1}, y_{n+1}) + f(t_n, y_n)\right), \qquad \text{(This is the trapezoidal rule)}$$

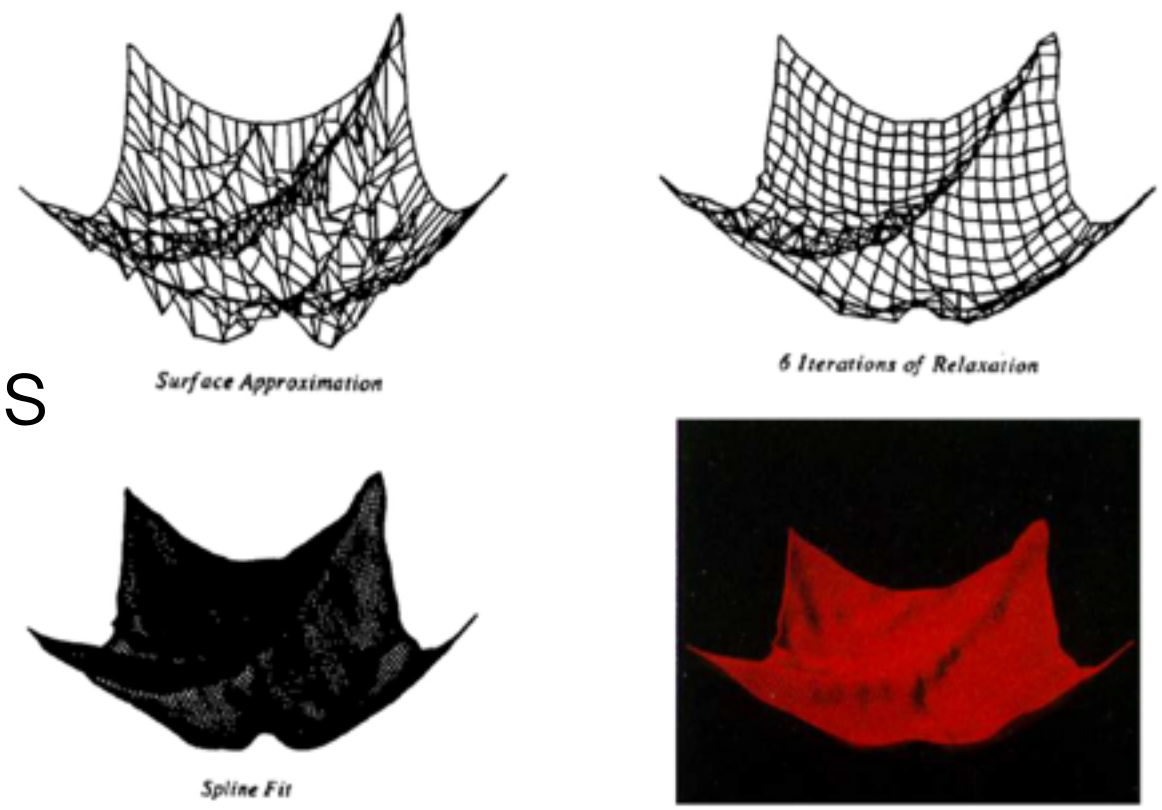
$$y_{n+2} = y_{n+1} + h\left(\frac{5}{12}f(t_{n+2}, y_{n+2}) + \frac{2}{3}f(t_{n+1}, y_{n+1}) - \frac{1}{12}f(t_n, y_n)\right),$$

$$y_{n+3} = y_{n+2} + h\left(\frac{3}{8}f(t_{n+3}, y_{n+3}) + \frac{19}{24}f(t_{n+2}, y_{n+2}) - \frac{5}{24}f(t_{n+1}, y_{n+1}) + \frac{1}{24}f(t_n, y_n)\right).$$

Jerry Weil (1986)

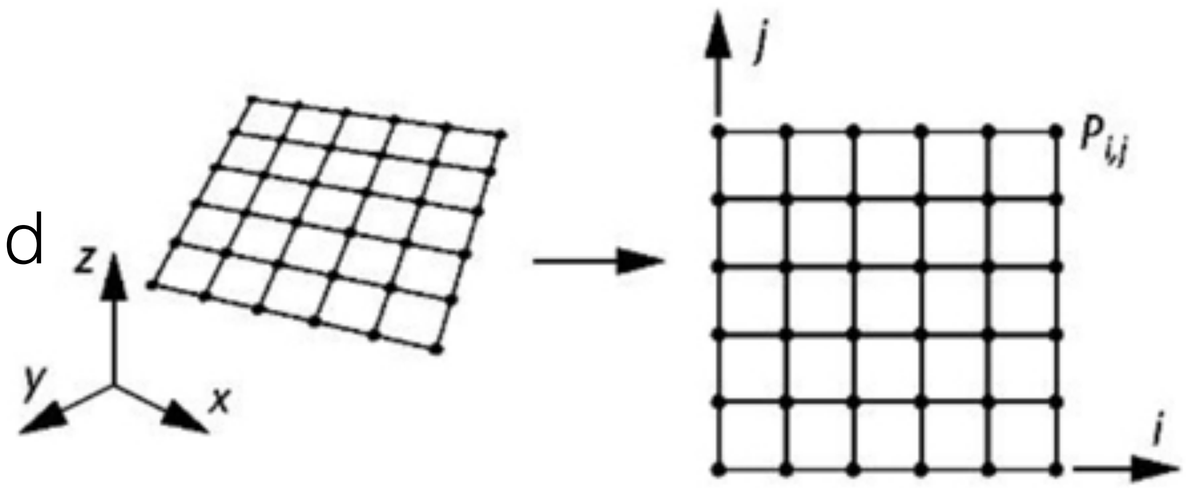
- Geometric model $y = a \cosh(x/b)$
  - A cable under self-weight forms a catenary curve at equilibrium
  - A cloth hanging from a discrete number of points can be described by a system of these curves
  - Limitation: only models the hanging clothes





Feynman (1986)

- Physically-based model
  - Represented cloth in a 3D space by using a 2D grid
  - Energy-based method:
    The energy for each point is calculated in relation to surrounding points
  - The final position of cloth was derived based on the minimization of energy



$$E(P_{i,j}) = k_s E_{elastic(i,j)} + k_b E_{bending(i,j)} + k_g E_{gravitational(i,j)}$$

  - Limitation: only modeling cloth draped over rigid objects

Breen, House, Wozny(1991-1994)

- Each particle is based on thread-level interactions

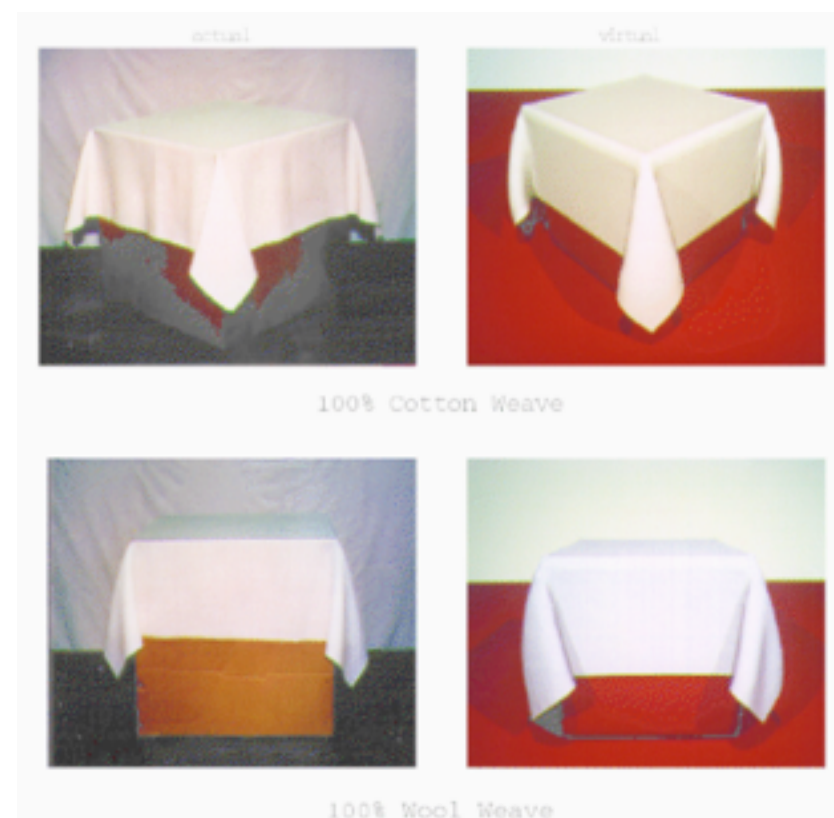
- Energy: Stretching, bending, trellising (shear) & gravity

$$E_{total_{ij}} = E_{repel_{ij}} + E_{stretch_{ij}} + E_{bend_{ij}} + E_{trellis_{ij}} + E_{gravity_{ij}}$$

- Minimize total energy (SGD), while maintaining collision constraints

- Fit functions to the measured data

- Limitation: No dynamics involved



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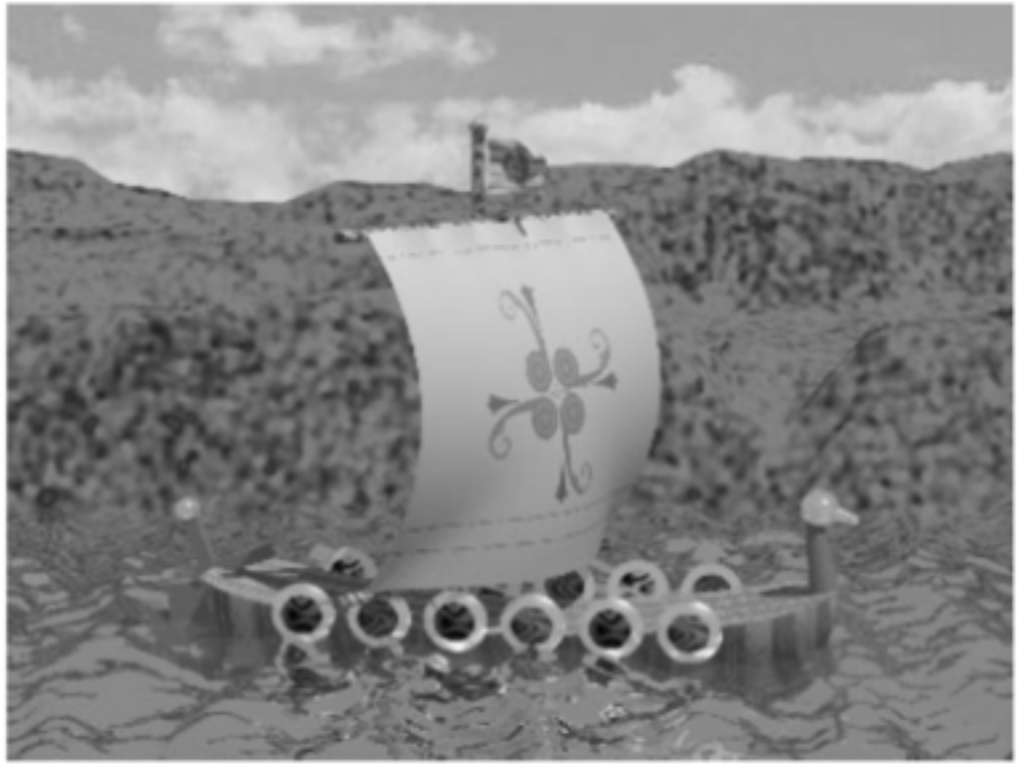
Provot (1995)

- Spring-Mass System
  - Internal Forces and External Forces
  - Integration: Simple Euler method
  - Dynamic Inverse Procedures



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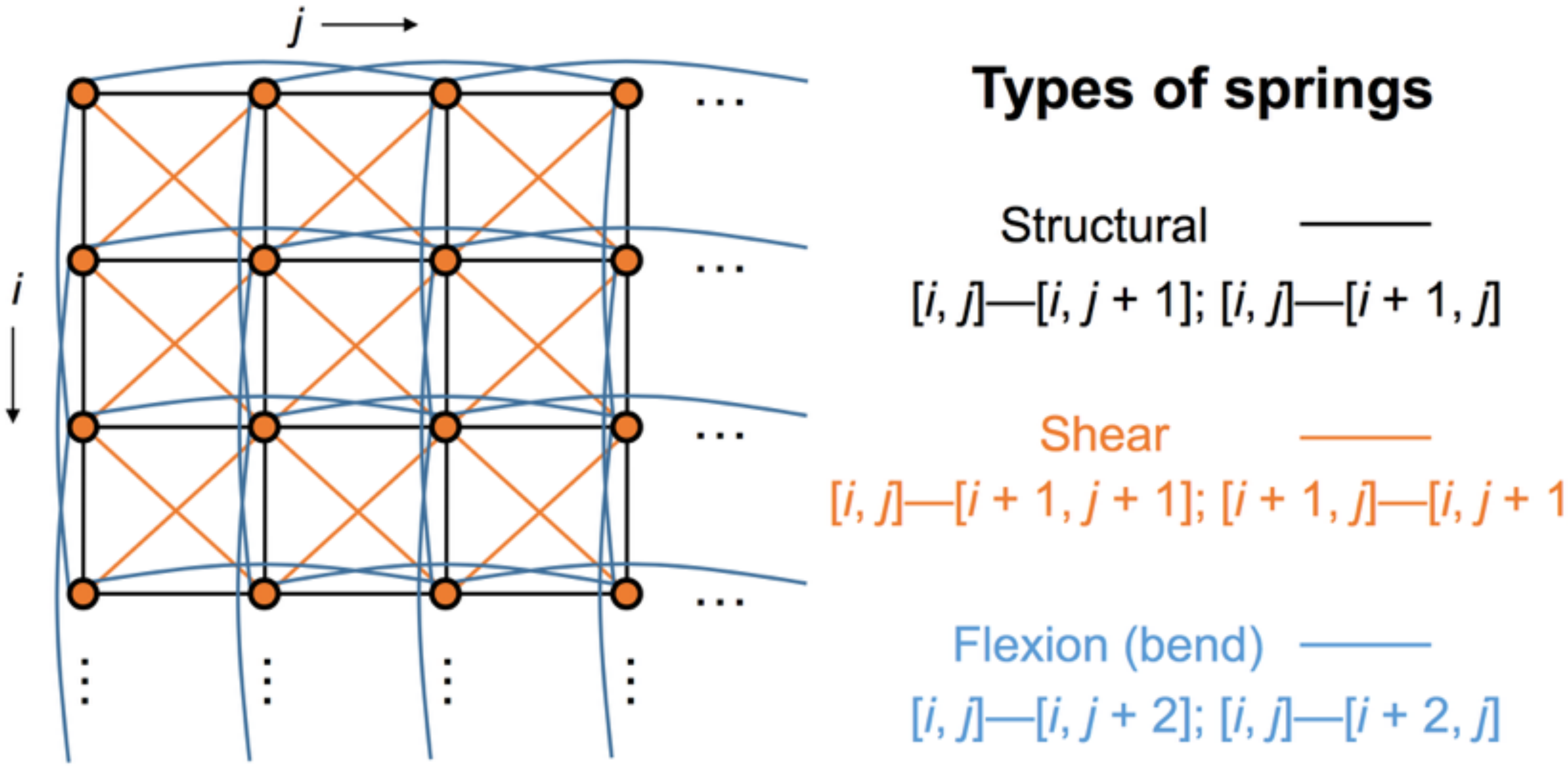
Haumann (1987) Extension

Spring-Mass System for Cloth

- Consider a rectangular cloth with m×n particles



Provot (1995)

Spring-Mass System for Cloth

- Internal Forces: F = - k·u

$$\mathbf{F}_{int}(P_{i,j}) =$$
$$-\sum_{(k,l)\in\mathcal{R}} K_{i,j,k,l}\left[\mathbf{l}_{i,j,k,l} - l^0_{i,j,k,l}\frac{\mathbf{l}_{i,j,k,l}}{\|\mathbf{l}_{i,j,k,l}\|}\right] \quad (1)$$

where:

- $\mathcal{R}$ is the set regrouping all couples (k,l) such as $P_{k,l}$ is linked by a spring to $P_{i,j}$,
- $\mathbf{l}_{i,j,k,l} = \overrightarrow{P_{i,j}P_{k,l}}$,
- $l^0_{i,j,k,l}$ is the natural length of the spring linking $P_{i,j}$ and $P_{k,l}$,
- $K_{i,j,k,l}$ is the stiffness of the spring linking $P_{i,j}$ and $P_{k,l}$.

Notations:

- The system is the mesh of m x n masses
- P_{i,j}(t): position at time t
- u: deformation (displacement from equilibrium) of the elastic body subjected to the force F

Provot (1995)

Spring-Mass System for Cloth

- External Forces
  - Force of gravity

$$F_{gr}(P_{i,j}) = \mu g$$

  - Viscous damping

$$F_{dis}(P_{i,j}) = -C_{dis} v_{i,j}$$

  - Viscous fluid (wind)

Notations:

- \mu: mass
- g: the acceleration of gravity
- C_{dis} : damping coefficient
- v_{i,j} : velocity at point P_{i,j}.

Provot (1995)

Spring-Mass System for Cloth

- External Forces
  - Viscous fluid (wind)

    Force: a viscous fluid moving at a uniform velocity u_{fluid} exerts, on a surface of a body moving at a velocity v

$$F_{vi}(P_{i,j}) = C_{vi}\,[n_{i,j} \cdot (u_{fluid} - v_{i,j})]\; n_{i,j}$$

- u_{fluid}: a viscous fluid with uniform velocity
- v_{i,j}: velocity at point P_{i,j}
- n_{i,j} is the unit normal at P_{i,j}
- C_{vi} is the viscosity constant

Provot (1995)

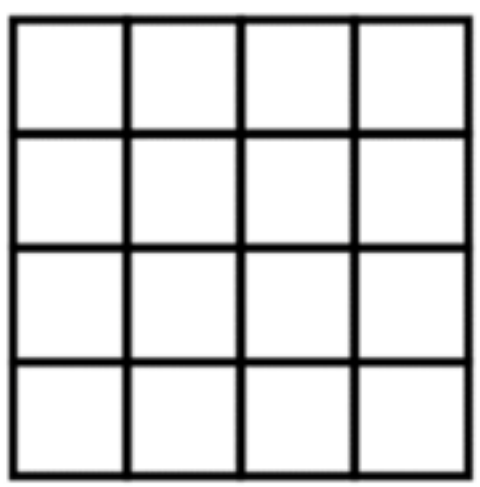
Spring-Mass System for Cloth

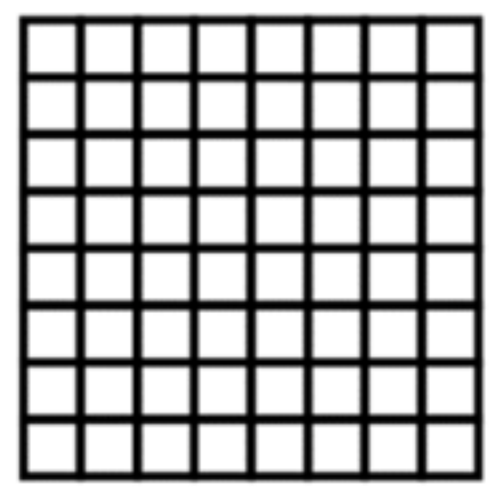
- Integration: Simple Euler Method
- Dynamic Inverse Procedure
  - movement is not entirely caused by analytically computed forces (Contact problems: hanging)
  - compute displacement due to the force =>
    we know displacement of a hanging point (=0), compute actual velocity and actual resulting force
  - can be used in object collisions and self-intersection

Provot (1995)

Spring-Mass System for Cloth

- Discretization Problem
  - Discretize our cloth more or less finely
  - It takes a lot of effort to design discretization-independent schemes.





Provot (1995)

Baraff and Witkin(1998)

- Triangle-based representation
- Exploit sparseness of Jacobian
- Implicit integration
- Result - larger time steps, faster simulations (a few CPU-secs/frame)
- Used in Maya Cloth

Large Steps in Cloth Simulation

$$\ddot{x} = M^{-1}(-\frac{\partial E}{\partial x} + F)$$

- Every particle has a changing position x_i
- Given a vector condition C(**x**) which we want to be zero
- Associate an energy function Ec with C , k is stiffness constant of our choice

$$E_{\mathbf{C}}(\mathbf{x}) = \frac{k}{2}\mathbf{C}(\mathbf{x})^T\mathbf{C}(\mathbf{x})$$

- Assuming that C depends on only a few particle, C gives rise to a sparse force vector f.

$$\mathbf{f}_i = -\frac{\partial E_{\mathbf{C}}}{\partial \mathbf{x}_i} = -k\frac{\partial \mathbf{C}(\mathbf{x})}{\partial \mathbf{x}_i}\mathbf{C}(\mathbf{x})$$

Baraff and Witkin(1998)

Large Steps in Cloth Simulation

$$\mathbf{f}_i = -\frac{\partial E_\mathbf{C}}{\partial \mathbf{x}_i} = -k \frac{\partial \mathbf{C}(\mathbf{x})}{\partial \mathbf{x}_i} \mathbf{C}(\mathbf{x})$$

- Derivative matrix $\mathbf{K} = \partial \mathbf{f} / \partial \mathbf{x}$
- Nonzero entries of K are K_{ij} for all pairs of particles i and j that C depends on

$$\mathbf{K}_{ij} = \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} = -k \left(\frac{\partial \mathbf{C}(\mathbf{x})}{\partial \mathbf{x}_i} \frac{\partial \mathbf{C}(\mathbf{x})}{\partial \mathbf{x}_j}^T + \frac{\partial^2 \mathbf{C}(\mathbf{x})}{\partial \mathbf{x}_i \partial \mathbf{x}_j} \mathbf{C}(\mathbf{x}) \right)$$

- K is symmetric. $\partial^2 E / \partial \mathbf{x}_i \partial \mathbf{x}_j$
- Also, K is sparse

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Baraff and Witkin(1998)

Large Steps in Cloth Simulation

Stretch Forces

- Stretch can be measured by

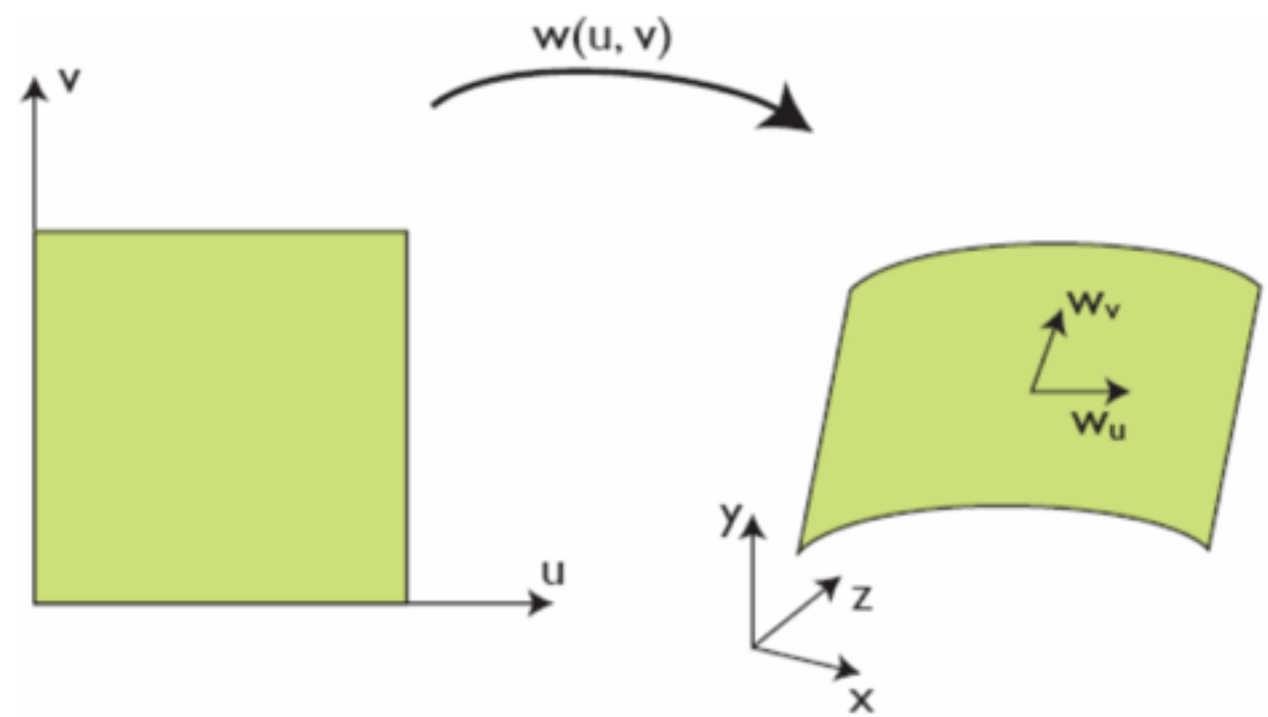
$$\mathbf{w}_u = \partial \mathbf{w} / \partial u \text{ and } \mathbf{w}_v = \partial \mathbf{w} / \partial v$$

- Material is unstretched wherever

$$\|\mathbf{w}_u\| = 1 \qquad \|\mathbf{w}_v\| = 1$$

- How to calculate?

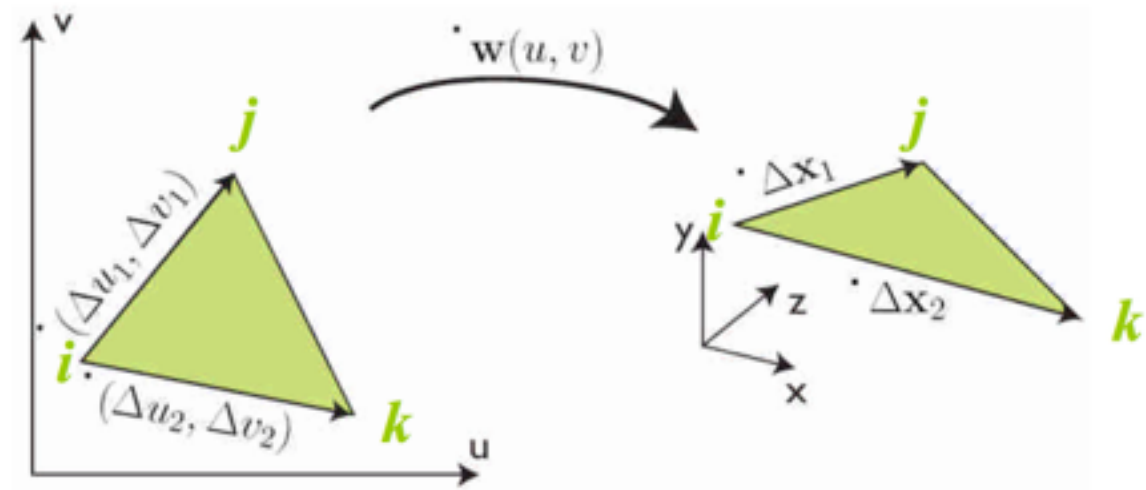
- Every cloth particle has
  - Changing position x_i in world space
  - Fixed plane coordinate (u_i , v_i)
- Suppose we have a single continuous function w(u, v) that maps from plane coordinates to world space



Baraff and Witkin(1998)

Large Steps in Cloth Simulation

Stretch Forces



- Stretch can be measured by

$$\mathbf{w}_u = \partial \mathbf{w}/\partial u \text{ and } \mathbf{w}_v = \partial \mathbf{w}/\partial v$$

- Material is unstretched wherever

$$\|\mathbf{w}_u\| = 1 \qquad \|\mathbf{w}_v\| = 1$$

$$\Delta x_1 = w_u \Delta u_1 + w_v \Delta v_1$$
$$\Delta x_2 = w_u \Delta u_2 + w_v \Delta v_2$$

- The condition for the stretch energy

$$\mathbf{C}(\mathbf{x}) = a \begin{pmatrix} \|\mathbf{w}_u(\mathbf{x})\| - b_u \\ \|\mathbf{w}_v(\mathbf{x})\| - b_v \end{pmatrix}$$

Solution：

$$(\mathbf{w}_u \quad \mathbf{w}_v) = (\Delta \mathbf{x}_1 \quad \Delta \mathbf{x}_2) \begin{pmatrix} \Delta u_1 & \Delta u_2 \\ \Delta v_1 & \Delta v_2 \end{pmatrix}^{-1}$$

a is the triangle’s area in uv coordinates

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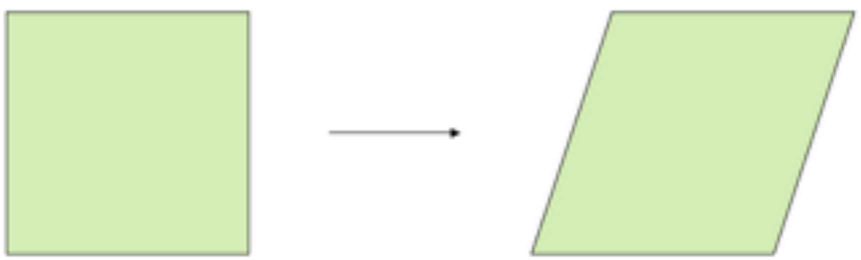
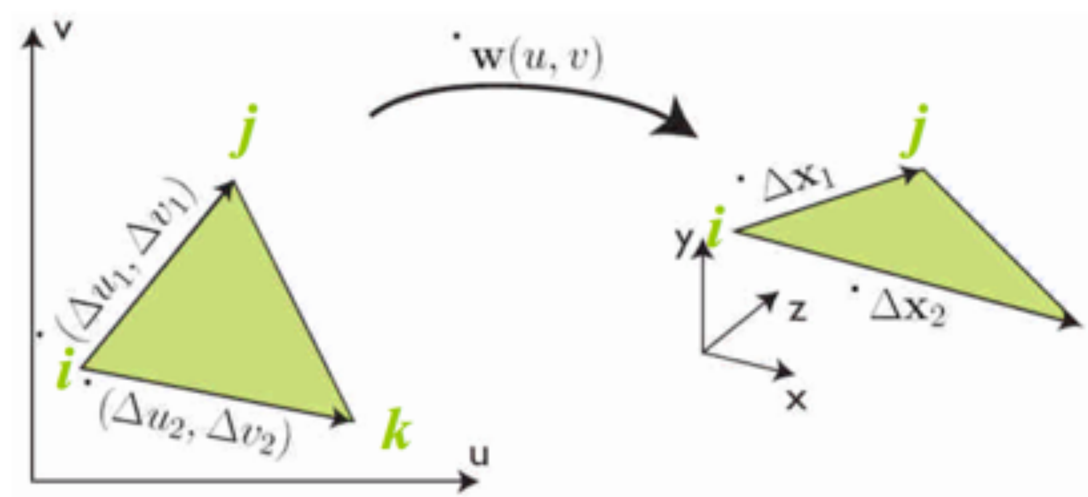
Baraff and Witkin(1998)

Large Steps in Cloth Simulation

Sheer Forces



Sheer



- Approximation to the shear angle

$$C(\mathbf{x}) = a\mathbf{w}_u(\mathbf{x})^T\mathbf{w}_v(\mathbf{x})$$

a the triangle's area in the uv plane.

$$\mathbf{w}_u = \partial\mathbf{w}/\partial u \text{ and } \mathbf{w}_v = \partial\mathbf{w}/\partial v$$

$$(\mathbf{w}_u \quad \mathbf{w}_v) = (\Delta\mathbf{x}_1 \quad \Delta\mathbf{x}_2)\begin{pmatrix}\Delta u_1 & \Delta u_2 \\ \Delta v_1 & \Delta v_2\end{pmatrix}^{-1}$$

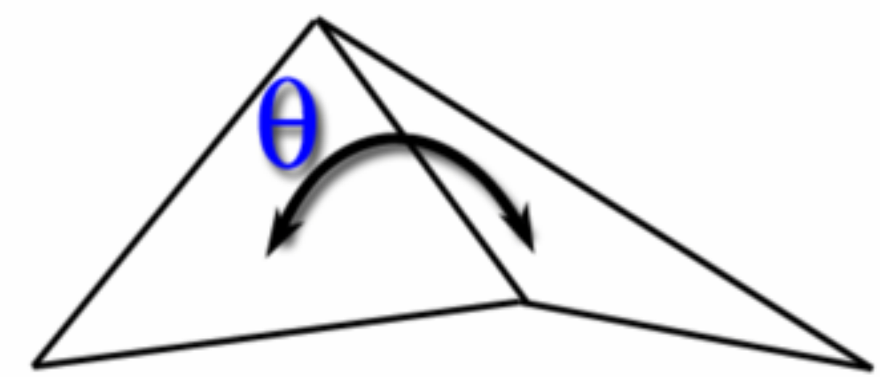
Baraff and Witkin(1998)

Large Steps in Cloth Simulation

Bend Forces

$$C(\mathbf{x}) = \theta$$

$$\sin\theta = (\mathbf{n}_1 \times \mathbf{n}_2) \cdot \mathbf{e} \text{ and } \cos\theta = \mathbf{n}_1 \cdot \mathbf{n}_2$$



bend

- n1 and n2 : the unit normals of the two triangles
- e: a unit vector parallel to the common edge

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Baraff and Witkin(1998)

Large Steps in Cloth Simulation

Damping Forces

$$E_{\mathbf{C}}(\mathbf{x}) = \frac{k}{2}\mathbf{C}(\mathbf{x})^T\mathbf{C}(\mathbf{x})$$

$$\mathbf{f}_i = -\frac{\partial E_{\mathbf{C}}}{\partial \mathbf{x}_i} = -k\frac{\partial \mathbf{C}(\mathbf{x})}{\partial \mathbf{x}_i}\mathbf{C}(\mathbf{x})$$

- The force f arising from the energy acts only in the direction $\partial \mathbf{C}(\mathbf{x})/\partial \mathbf{x}$.
- So should the damping force
- damping force should depend on the component of the system's velocity in $\partial \mathbf{C}(\mathbf{x})/\partial \mathbf{x}$ direction
- So the damping strength should depend on $(\partial \mathbf{C}(\mathbf{x})/\partial \mathbf{x})^T\dot{\mathbf{x}} = \dot{\mathbf{C}}(\mathbf{x})$

$$\mathbf{d} = -k_d\frac{\partial \mathbf{C}(\mathbf{x})}{\partial \mathbf{x}}\dot{\mathbf{C}}(\mathbf{x}).$$

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Baraff and Witkin(1998)

Large Steps in Cloth Simulation

Constraints

Constraints determined by the user or contact constraints

- Reduced Coordinates
- Penalty Methods
- Lagrange Multipliers

Enforcing constraints by mass modification

Example: zero acceleration along z-axis

$$\ddot{\mathbf{x}}_i = \begin{pmatrix} 1/m_i & 0 & 0 \\ 0 & 1/m_i & 0 \\ 0 & 0 & 0 \end{pmatrix} \mathbf{f}_i$$

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Baraff and Witkin(1998)

Large Steps in Cloth Simulation

Solving Equations

- Resultant sparse linear system
    - solved using conjugate gradient
- Integration:
    - Backward Euler (implicit method)
- Adaptive time stepping

Baraff and Witkin(1998)

Large Steps in Cloth Simulation

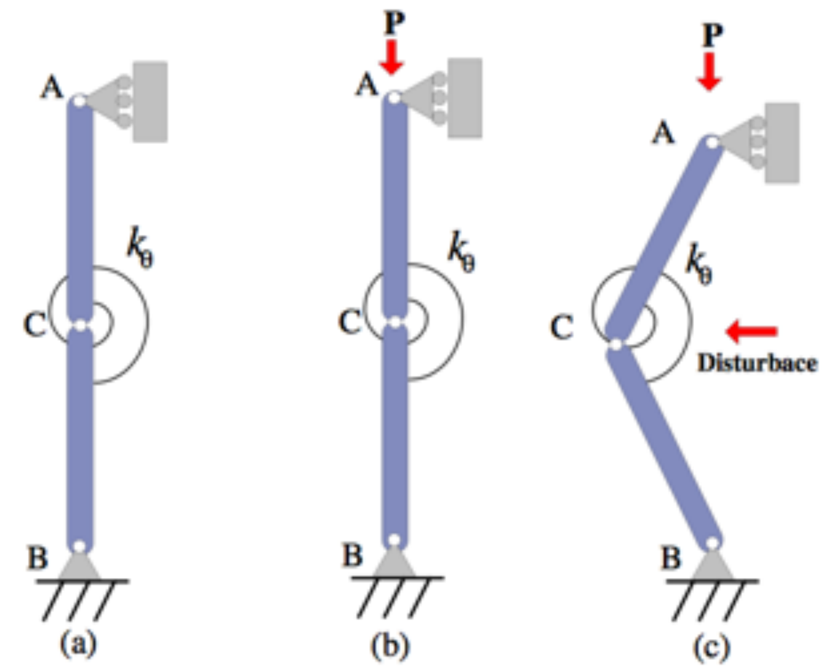
Collision

- Collision detection:
  - cloth-cloth: particle-triangle and edge-edge intersection
  - cloth-solid: cloth particle against the faces of solid object
- Collision Response:
  - cloth-cloth: Insert a strong damped spring force to push the cloth apart
  - cloth-solid: If the relative tangential velocity is low, lock the particle onto the surface; If not allow the particle to slide on the surface.

Baraff and Witkin(1998)

Choi and Ko (2002)

- Cloth property
  - Weak resistance to bending
  - Strong resistance to tension
- Need large compression forces for out-of-plane motion



- Use column buckling as their basic model
- Replace bend and compression forces with a single nonlinear model
- Semi-implicit cloth simulation technique (BDF2)
- Allows a large fixed time step

Stable but Responsible Cloth

Bridson, Marino, Fedkiw(2003)

- Clothing with many folds and wrinkle
- Accurate Model for Bending :
    - possibly nonzero rest angles for modeling wrinkles into the cloth
- Mixed explicit/implicit integration (Crank-Nicolson)
- Collisions: Forecasting collision response technique that promotes the development of detail in contact regions. Post-processing method for treating cloth-character collisions that preserves folds and wrinkles
- Dynamic constraint mechanism that helps to control large scale folding

Figure 5: *Wrinkles and folds in this CG cloth from Terminator 3: Rise of the Machines are preserved even when tightly stretched over a level set collision volume.*

English and Bridson (2008)

- Effective new discretization for deformable surfaces
- Constrained to not deform at all in-plane but free to bend out-of-plane
  - A triangle is rigid if and only if the distance between any two edge midpoints remains constant

$$c_{ij}(x) = \|x_i - x_j\|^2 - d_{ij}^2 = 0$$

  - Lagrange multiplier constraint forces

$$F_c = \left(\frac{\partial C}{\partial x}\right)^T \lambda = J^T \lambda,$$

- Second order accurate multistep constrained mechanics time integration scheme (BDF2)

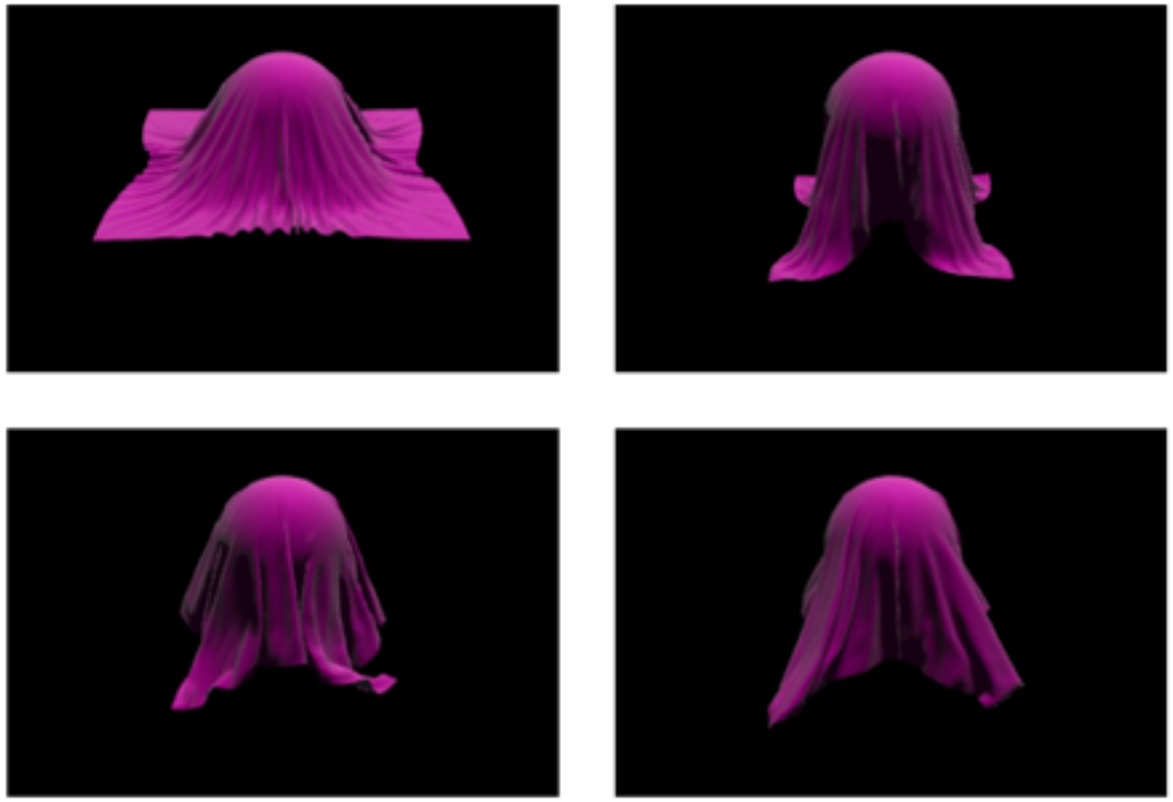


Figure 5: *A developable surface is dropped on a sphere, with immediate wrinkling and creasing patterns.*

References

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  - SIG GRAPH 2003 Course: Clothing Simulation and Animation
  - SIG GRAPH 2005 Course: Advanced Topics Clothing Simulation and Animation
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